AD-A065 631 PURDUE UNIV LAFAYETTE IND SCHOOL OF AERONAUTICS AND --ETC F/G 11/4
EFFECT OF HIGH LOAD ON THE FATIGUE BEHAVIOR OF UNNOTCHED GRAPHI--ETC(U)
NOV 78 C T SUN, J N YANG F33615-77-C-5123

UNCLASSIFIED AFML-TR-78-147 NL

| OF |

AD
A065631

END
DATE
FILMED
5-79
DDC

AD A0 65631

LEVEL II

(12)

AFML-TR-78-147

EFFECT OF HIGH LOAD ON THE FATIGUE BEHAVIOR OF UNNOTCHED GRAPHITE/EPOXY LAMINATES

PURDUE UNIVERSITY
WEST LAFAYETTE, INDIANA 47907

NOVEMBER 1978

TECHNICAL REPORT AFML-TR-78-147
Final Report — June 1977 - May 1978

Approved for public release; distribution unlimited.

AIR FORCE MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (ASD/OIP) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

Stephen W. Tsai

STEPHEN W. TSAI
Project Scientist

FOR THE COMMANDER

JEROME M. KELBLE, Chief
Nonmetallic Materials Division

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: AFML-TR-78-147	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): EFFECT OF HIGH LOAD ON THE FATIGUE BEHAVIOR OF UNNOTCHED GRAPHITE/EPOXY LAMINATES.		5. TYPE OF REPORT & PERIOD COVERED: Final Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): C. T. Sun, J. N. Yang		8. CONTRACT OR GRANT NUMBER(s): F33615-77-C-5123
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Purdue University, School of Aeronautics & Astronautics, West Lafayette, Indiana 47907		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 2419-03-05
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Materials Laboratory (AFML/MBM), Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio 45433		12. REPORT DATE: Nov 1978
		13. NUMBER OF PAGES: 45
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report): UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Final Rept. Jun 77 - May 78

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composites
Graphite/epoxy
Fatigue
Residual Strength Degradation
Effect of High Load

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The purpose of this research program is to verify experimentally the theoretical prediction for the effect of the high load on the fatigue behavior of unnotched composite laminates. A three-parameter fatigue and residual strength degradation model has been used to derive the number of load cycles a specimen can survive and the statistical distributions of the fatigue life and the residual strength after the specimen has survived an initial high load or periodic high loads. One set of the static ultimate strength data and one over

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

291850

79 03 13 020

set of the fatigue scan data were generated to determine the three-parameter values appearing in the theoretical model. It is confirmed by the present test program that a specimen is guranteed to survive a certain number of load cycles as predicted by the theoretical model, after the specimen has survived a high load prior to the application of fatigue loading. The correlation between the three-parameter theoretical model and the test results is reasonable, while the correlation between the theoretical prediction for the effect of the high load and the test results is acceptable. It is concluded that the three-parameter fatigue and residual strength degradation model is capable of predicting the gross fatigue behavior of unnotched composite laminates as well as the effect of the high load on the subsequent fatigue behavior. The theoretical model seems to offer a simple and resonable approach for the preliminary fatigue analysis of unnotched composites.

FOREWORD

This report describes an investigation of the effect of the high load on the fatigue behavior of unnotched graphite/epoxy composite laminates for structural applications performed by the Purdue University under Air Force Contract F33615-77-C-5123. Pertinent experimental tests were carried out at the Purdue University, and the theoretical development and analysis of data were conducted at the George Washington University. The Air Force Project Engineer directing the program was Dr. Steve Tsai of the Mechanics and Surface Interaction Branch, Air Force Materials Laboratory at Wright-Patterson AFB, Ohio. Dr. C. T. Sun of Purdue University was the principal investigator and Dr. J. N. Yang of The George Washington University was the co-principal investigator.

TABLE OF CONTENTS

[redacted]

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION I

INTRODUCTION

A limited amount of test data indicates that when a composite specimen survives a static high load, it will survive at least a certain number of fatigue cycles [Refs. 1-3]. This characteristic possesses an important practical application, in the sense that a finite service (fatigue) life can be guarranteed for those composites that survive a high load prior to service [Refs. 1-3]. Moreover, the service life can further be extended by applying a prescribed level of high loads to the composites at periodic intervals in service [Refs. 3-4].

The theoretical models for predicting the effect of the high load on the fatigue behavior of composites have been discussed in Refs. 1-4. A two-parameter model has been proposed in Refs. 1-2, while a three-parameter model has been derived in Ref. 3 along with the effect of periodic high loads.

It is the purpose of this research investigation to experimentally verify the theoretical model for the effect of the high load on the fatigue behavior of composite laminates proposed in Ref. 3. The test program includes only unnotched coupons in room temperature, laboratory air environment.

The theoretical approach to predict the effect of the high load on the fatigue behavior of composites is based on the fatigue and residual strength degradation model [Refs. 1-13], on the premise that the unnotched specimen which is stronger in

static strength will be stronger in fatigue. The three-parameter fatigue and residual strength degradation model derived in Ref. 3 was extended in Ref. 5 to account for the effect of compressive loads. Such a theoretical model was verified in Ref. 3 by a limited amount of test data and it was further verified in Ref. 5 by use of extensive test results generated in Ref. 6. Since the research effort presented in Ref. 5 is supported by the subject contract, it is presented in the Appendix. The three-parameter fatigue and residual strength degradation model [Ref. 5] is further verified by the test results obtained in the present experimental program.

The main advantage of the three-parameter fatigue and residual strength degradation model is that the three parameters c,b and K appearing in the model can be determined from a limited amount of test data. Only one set of the ultimate strength data and one set of the fatigue scan data are sufficient to determine the values of c,b and K. Once the parameter values c,b and K have been determined, the theoretical model is capable of predicting the statistical distributions of both the fatigue life (under any cyclic stress level) and the residual strength (for any number of load cycles). Furthermore, without any additional test data, the theoretical model can predict the effect of the high load on the fatigue behavior of composite laminates, including the minimum number of load cycles a specimen can survive after it has survived a high load, and the statistical distributions of the fatigue life and the residual strength for a

specimen subjected to both the initial high load (or periodic high loads) and the fatigue loading.

One set of 15 ultimate strength data and one set of 39 fatigue scan data have been generated to determine the three parameter values appearing in the theoretical model. Then, four sets of test data on the fatigue life and the residual strength, two with an initial high load and another two without it, have been generated to verify the validity of the three-parameter fatigue and residual strength degradation model as well as the validity of the theoretical predictions for the effect of the high load on the fatigue behavior of unnotched composites.

It is shown that the correlation between the theoretical predictions and the test results on the statistical distribution of the fatigue life is excellent. The correlation between the theoretical predictions and test results on the statistical distribution of the residual strength is reasonable although a small discrepency does exist. It is confirmed by our test program that when a specimen survives an initial high load, it can survive a certain number of load cycles as predicted from the theoretical model.

In light of the complexity of the fatigue process involved, the three-parameter fatigue and residual strength degradation model appears to be useful in predicting the gross fatigue behavior of unnotched composite laminates as evidenced by the extensive amount of test data.

SECTION II

THEORETICAL MODEL

1. FATIGUE AND RESIDUAL STRENGTH DEGRADATION MODEL

A three-parameter fatigue and residual strength degradation model for unnotched composite laminates under the cyclic loading has been derived in Refs. 5 (Eqs. 14 and 15) and 3 (Eqs. 13 and 14) as follows

$$R^c(n_1) = R^c(n_0) - \beta^c K S^b (n_1 - n_0) \tag{1}$$

in which $R(n_1)$ and $R(n_0)$ are the residual strengths at n_1 and n_0 cycles ($n_1 > n_0$), respectively, β is the scale parameter of the ultimate strength, b,c and K are three parameters to be determined from the test data, and S is the stress range defined as

$$S = \sigma_{max} - \sigma_{min} \tag{2}$$

where σ_{max} and σ_{min} represent the maximum and the minimum cyclic stresses, respectively.

For $n_0=0$ and $n_1=n$, Eq. 1 reduces to the following;

$$R^c(n) = R^c(0) - \beta^c K S^b n \tag{3}$$

in which R(0) is the ultimate strength.

Eqs. 1 and 3 indicate that the residual strength decays monotonically with respect to the number of fatigue cycles. The

ultimate strength R(0) is a statistical variable assumed to follow the two-parameter Weibull distribution,

$$F_{R(0)}(x) = P[R(0) \leq x] = 1-\exp[-(x/\beta)^{\alpha}] \tag{4}$$

in which $F_{R(0)}(x)$ is the probability that the ultimate strength is smaller than a value x, α is the shape parameter, and β is the scale parameter (or the characteristic strength). Both α and β should be determined from the test results of the ultimate strength.

2. STATISTICAL DISTRIBUTION OF FATIGUE LIFE AND RESIDUAL STRENGTH

It is assumed that the fatigue failure occurs when the residual strength R(n) has been decayed to the value equal to the maximum cyclic stress σ_{max}. Let N be the fatigue life denoting the number of cycles to failure under the constant amplitude cyclic loading. Then, the fatigue fracture takes place when

$$R(n) = \sigma_{max} \quad ; \quad N = n \tag{5}$$

Substitution of Eq. 5 into the residual strength degradation model, Eq. 3, leads to an expression for the fatigue life N in the form

$$N = [R^{c}(0)-\sigma_{max}^{c}]/\beta^{c}KS^{b} \tag{6}$$

in which N is a statistical variable, since the ultimate strength R(0) is a statistical variable.

The statistical distribution of the fatigue life, N, can be obtained from that of the ultimate strength, R(0), given by Eq. 4, through the transformation of Eq. 6 as follows;

$$F_N(n) = P[N \leq n] = P[R(0) \leq (n\beta^c KS^b + \sigma_{max}^{\ c})^{1/c}]$$

$$= 1-\exp\left\{-\left[\frac{n+(\sigma_{max}^{\ c}/\beta^c KS^b)}{1/KS^b}\right]^{\alpha/c}\right\} \tag{7}$$

The distribution function of the fatigue life represented by Eq. 7 is a three-parameter Weibull distribution with a lower bound at $-\sigma_{max}^{\ c}/\beta^c KS^b$. The negative lower bound comes from the fact that the ultimate strength is assumed to follow the two-parameter Weibull distribution (lower bound at zero) so that there is a finite probability of initial failure when the ultimate strength is smaller than the maximum applied stress.

The initial failure probability, i.e., the probability that the ultimate strength R(0) is smaller than σ_{max}, can be obtained from Eq. 7 by setting n=0,

$$F_N(0) = 1-\exp[-(\sigma_{max}/\beta)^{\alpha}] \tag{8}$$

It can be observed from Eqs. 4 and 8 that

$$F_N(0) = F_{R(0)}(\sigma_{max}) = P[R(0) \leq \sigma_{max}]$$

as expected.

Such an initial failure with a finite probability is a discrete event representing a sudden jump in the distribution

function from zero to $F_N(0)$, Eq. 8, at n=0. As a result, the distribution function of the fatigue life follows from Eq. 7 as

$$F_N(n) = 0 \qquad ; \quad n \leq 0$$

$$= 1-\exp\left\{-\left[\frac{n+(\sigma_{max}^c/\beta^c KS^b)}{1/KS^b}\right]^{\alpha/c}\right\}; \quad n>0 \tag{9}$$

The statistical distribution of the residual strength, R(n), at n fatigue cycles can be obtained from that of the ultimate strength, R(0), given by Eq. 4, through the transformation of Eq. 3 as follows;

$$F_{R(n)}(x) = P[R(n) \leq x] = P[R^c(0) - \beta^c KS^b n \leq x^c]$$

$$= 1-\exp\left\{-\left[\frac{x^c+\beta^c KS^b n}{\beta^c}\right]^{\alpha/c}\right\}; \quad x \geq \sigma_{max} \tag{10}$$

The probability that the residual strength R(n) is smaller than σ_{max}, i.e., the probability that the specimen will fail before n cycles of load application, follows from Eq. 10 as

$$F_{R(n)}(\sigma_{max}) = P[R(n) \leq \sigma_{max}]$$

$$= 1-\exp\left\{-\left[\frac{\sigma_{max}^c+\beta^c KS^b n}{\beta^c}\right]^{\alpha/c}\right\} \tag{11}$$

Under n cycles of load application, the specimen may fail with a probability given by Eq. 11. Once the specimen fails

before n fatigue cycles, the residual strength at n cycles becomes null. As a result, the distribution function given by Eq. 10 is restricted to the region where $x \geq \sigma_{max}$ as indicated.

Let $R^*(n)$ be the residual strength for those specimens which have survived n cycles of load application, i.e., $R^*(n)$ is always greater than σ_{max}. Then, the distribution function of $R^*(n)$ can be derived as follows;

$$\begin{aligned} F_{R^*(n)}(x) &= 1-P[R^*(n) \geq x] = 1-P[R(n) \geq x \,|\, R(n) \geq \sigma_{max}] \\ &= 1-\Big\{P[R(n) \geq x]/P[R(n) \geq \sigma_{max}]\Big\} \;;\quad x \geq \sigma_{max} \end{aligned} \tag{12}$$

Substitution of Eqs. 10 and 11 into Eq. 12 then yields

$$F_{R^*(n)}(x) = 1-\exp\left\{\left[\frac{\sigma_{max}^{\,c}+\beta^c K S^b n}{\beta^c}\right]^{\alpha/c} - \left[\frac{x^c+\beta^c K S^b n}{\beta^c}\right]^{\alpha/c}\right\};\; x \geq \sigma_{max} \tag{13}$$

for the residual strength distribution of the surviving specimens.

It will be shown later that if the quality of the composite specimens is poor a sampling fluctuation will always occur. Consequently, if the probability of the fatigue failure before n cycles of load application is small, the test results may deviate from the theoretical prediction, $F_{R(n)}(\sigma_{max})$ given by Eq. 11, significantly, thus altering the shape of the distribution function $F_{R^*(n)}(x)$ considerably. Under such a situation, Eq. 10, which accounts for the entire population of the test specimens, should be used.

3. EFFECT OF HIGH LOAD

When a high load is applied to the specimen prior to the fatigue loading as shown in Fig. 1(a), the effect of the high load on the subsequent fatigue life and the residual strength of the specimen has been discussed in Refs. 1-4. Such an effect will be derived herein based on the three-parameter fatigue and residual strength degradation model given by Eq. 3.

Let r_0 be the level of high load applied to the specimens prior to the fatigue loading. Then, the probability that the specimen will fail under r_0, denoted by B_0, follows from Eq. 4 as

$$B_0 = P[R(0) \leq r_0] = 1 - \exp[-(r_0/\beta)^{\alpha}] \tag{14}$$

If R(0+) denotes the ultimate strength of the population for those specimens that survive r_0, then the statistical distribution of R(+0) can be obtained from that of R(0) as follows;

$$\begin{aligned} F_{R(0+)}(x) &= P[R(0+) \leq x] = 1 - P[R(0+) \geq x] \\ &= 1 - P[R(0) \geq x \mid R(0) \geq r_0] \qquad (15) \\ &= 1 - \{P[R(0) \geq x] / P[R(0) \geq r_0]\} \; ; \quad x \geq r_0 \end{aligned}$$

Substitution of Eqs. 4 and 14 into Eq. 15 leads to the distribution function of R(0+) as follows:

$$F_{R(0+)}(x) = P[R(0+) \leq x] = 1 - \exp\left\{(r_0/\beta)^{\alpha} - (x/\beta)^{\alpha}\right\} \; ; \quad x \geq r_0 \tag{16}$$

When a specimen survives the high load r_0, its ultimate strength is greater than or equal to r_0 under the assumption that no damage occurs due to the application of r_0. Let N_0 denote the minimum number of fatigue cycles that a specimen, after having survived r_0, can sustain. Then, N_0 can be obtained by substituting the condition of fatigue fracture (Eq. 5)

$$R(n) = \sigma_{max} \ , \quad n = N_0 \ , \quad R(0) = r_0 \tag{17}$$

into the fatigue and residual strength degradation model, Eq. 3, to yield

$$\sigma_{max}^{c} = r_0^c - \beta^c K S^b N_0$$

or

$$N_0 = [(r_0/\beta)^c - (\sigma_{max}/\beta)^c]/KS^b \tag{18}$$

Eq. 18 indicates that the minimum number, N_0, of load cycles the specimen is guarranteed to survive, after having survived the initial high load r_0, depends on r_0, and that N_0 increases as the initial high load r_0 increases.

The theoretical model given by Eq. 3 can be used for the population which has survived the initial high load r_0,

$$R^c(n) = R^c(0+) - \beta^c K S^b n \tag{19}$$

in which $R(n)$ is the residual strength after n cycles of load application for the population which has survived r_0.

Since R(0+) has a lower bound at r_0, the minimum residual strength after n cycles of load application, denoted by R_{min}, follows from Eq. 19 as

$$R_{min} = \left(r_0^c - \beta^c K S^b n\right)^{1/c} \tag{20}$$

The fatigue life N is obtained from Eq. 19 by applying the condition of fatigue fracture given by Eq. 5; with the result,

$$N = [R^c(0+) - \sigma_{max}^{\ c}]/\beta^c K S^b \tag{21}$$

The statistical distribution of the fatigue life N can be obtained from that of R(0+) represented by Eq. 16 through the transformation of Eq. 21 as follows;

$$\begin{aligned} F_N(n) &= P[N \le n] = P[R(0+) \le (n\beta^c K S^b + \sigma_{max}^{\ c})^{1/c}] \\ &= 1-\exp\left\{(r_0/\beta)^\alpha - \left[\frac{n+(\sigma_{max}^{\ c}/\beta^c K S^b)}{1/KS^b}\right]^{\alpha/c}\right\}; \quad n \ge N_0 \end{aligned} \tag{22}$$

in which N_0 is given by Eq. 18.

Furthermore, the statistical distribution of the residual strength R(n) after n cycles of load application for the population that aurvives r_0 can be obtained from that of R(0+) through the transformation of Eq. 19,

$$\begin{aligned} F_{R(n)}(x) &= P[R(n) \le x] = P[R^c(0+) - \beta^c K S^b n \le x^c] \\ &= P[R(0+) \le (x^c + \beta^c K S^b n)^{1/c}] \end{aligned}$$

Application of Eq. 16 into the above equation then yields

$$F_{R(n)}(x) = 1\text{-exp}\left\{(r_0/\beta)^{\alpha} - \left[\frac{x^c+\beta^c KS^b n}{\beta^c}\right]^{\alpha/c}\right\} \quad (23)$$

for the distribution function of the residual strength.

4. DETERMINATION OF MODEL PARAMETERS

The theoretical fatigue and residual strength degradation model represented by Eq. 1 or 3 involves three parameters, i.e., c, b and k, which should be determined from the test data. After these three parameter values have been determined, the theoretical model is capable of predicting (i) the statistical distribution of the fatigue life under any level of constant amplitude cyclic stress, σ_{max} and S, as given by Eq. 9, (ii) the statistical distribution of the residual strength for any number of cycles of load application, as shown by Eq. 10, (iii) the minimum number N_0 of fatigue cycles a specimen can survive after having survived a high load r_0 as derived in Eq. 18, and (iv) the statistical distributions of the fatigue life and the residual strength when a specimen is subjected to a high load r_0 followed by the fatigue loading as given by Eqs. 22 and 23, or when a specimen is subjected to both periodic high loads and the fatigue loading [see Eqs. 31,33 and Ref. 4].

As a result, only those test data which are needed for the determination of c,d and K should be obtained experimentally. An efficient analysis technique to minimize the number of test data required for the determination of c,b and K has been proposed in Refs. 3 and 5. It is plausible to mention that only

one set of the static ultimate strength data (usually 15-25 specimens) and one set of the fatigue scan data (usually 30-40 specimens) are sufficient for the determination of c, b and K. In addition, the fatigue scan data also provide valuable information for the gross fatigue behavior of composite laminates. The analysis and the test procedure are described in the following:

(i) A set of m specimens is tested statically to obtain the ultimate strength, dented by $(x_1, x_2, \ldots x_m)$ (see Table 1). The first three central moments of the ultimate strengths can easily be computed as

$$m_1 = \mu = \frac{1}{m} \sum_{i=1}^{m} x_i \ , \ m_2 = \frac{1}{m} \sum_{i=1}^{m} (x_i - \mu)^2 \ , \ m_3 = \frac{1}{m} \sum_{i=1}^{m} (x_i - \mu)^3 \quad (24)$$

Since the distribution of the ultimate strength is approximated by the two-parameter Weibull dsitribution, Eq. 4, the shape parameter α and the scale parameter β can easily be estimated from Eq. 24.

(ii) A set of J specimens is then subjected to the constant amplitude cyclic loading, and the results are referred to as the fatigue scan data [see Table 2]. Each specimen may be subjected to a different stress amplitude, and the number of different stress amplitudes used for testing the entire J specimens is preferable to cover the entire fatigue range of interest. Some specimens may be tested until fatigue fracture and others may be tested for a certain number of load cycles and then their residual strengths are measured. When the stress

amplitude is small, the specimen may not fail within 10^6 or 10^7 cycles, in which case the residual strength should be measured in order to save the testing time. Thus, the fatigue scan data consists of the fatigue life data and the residual strength data, both of which are equally useful.

Hence, the fatigue scan data can be denoted by $(R_1, S_1, \sigma_{max1}, n_1), \ldots, (R_i, S_i, \sigma_{maxi}, n_i), \ldots, (R_J, S_J, \sigma_{maxJ}, n_J)$ in which the ith fatigue scan data, $(R_i, S_i, \sigma_{maxi}, n_i)$, indicates that the ith specimen is subjected to a maximum cyclic stress σ_{maxi} with a stress range S_i for n_i cycles of load application and its residual strength is R_i. If the specimen fails in fatigue at n_i cycles, then $R_i = \sigma_{maxi}$, since the residual strength at the moment of fatigue fracture is equal to the maximum cyclic stress. All the information on $(R_i, S_i, \sigma_{maxi}, n_i)$, for $i=1,2,\ldots,J$, are obtained after the experimental tests for the fatigue scan data are finished.

The residual strength of each fatigue scan data $R_i (i=1,2,\ldots,J)$ can be converted into the equivalent static ultimate strength, denoted by $R_i(0) (i=1,2,\ldots,J)$, using the theoretical model given by Eq. 3,

$$R_i^c(0) = R_i^c + \beta^c K S_i^b n_i \tag{25}$$

in which R_i, S_i and n_i correspond to the ith fatigue scan data [see Table 2]. The equivalent ultimate strength $R_i(0)$ represents the ultimate strength of the ith specimen if it were tested statically until fracture.

The first three central moments of the equivalent ultimate strength obtained from the fatigue scan data are given by

$$\mu_1 = \frac{1}{J}\sum_{i=1}^{J} R_i(0), \quad \mu_2 = \frac{1}{J}\sum_{i=1}^{J} [R_i(0)-\mu_1]^2, \quad \mu_3 = \frac{1}{J}\sum_{i=1}^{J} [R_i(0)-\mu_1]^3 \quad (26)$$

Theoretically, if the number m of the ultimate strength data and the number J of the fatigue scan data are very large, the statistical distribution of x_i (i=1,2,...m) should be identical to that of $R_i(0)$ (i=1,2,...,J) and hence Eq. 24 should be identical to Eq. 26, i.e., $m_1=\mu_1$, $m_2=\mu_2$, and $m_3=\mu_3$. Since, however, m and J are finite it is not possible to match Eq. 24 to Eq. 26 in order to determine c,b and K. As a result, c,b and K are obtained by minimizing the mean square difference Δ of the three central moments, i.e.,

$$\Delta = (m_1-\mu_1)^2+g_1(\sqrt{m_2}-\sqrt{\mu_2})^2+g_3(\sqrt[3]{m_3}-\sqrt[3]{\mu_3})^2 \quad (27)$$

in which g_1 and g_2 are assigned weighting values to indicate the importance of matching the mean (first term), the standard deviation (the second term), and the skewness (the third term).

5. PERIODIC HIGH LOADS

When the specimen is subjected to both the cyclic loading and a high load r_0 that is applied at periodic intervals of T fatigue cycles as shown in Fig. 1(b), the statistical distribution function of the residual strength, R(jT), right after the

application of the j+1th high load, denoted by $F_{R(jT)}(x)$, has been derived in Ref. 4 [see Eq. 43] as

$$F_{R(jT)}(x) = P[R(jT) \leq x]$$

$$= 1-\exp\left\{\left(\frac{r_0^c+\Phi jT}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c+\Phi jT}{\beta^c}\right)^{\alpha/c}\right\} \tag{28}$$

in which

$$\Phi = \beta^c KS^b \tag{29}$$

The residual strength at jT+n cycles, denoted by R(jT+n), can be expressed in terms of the residual strength R(jT) using the theoretical model of Eq. 1 as follows:

$$R^c(jT+n) = R^c(jT)-\Phi n \tag{30}$$

in which n_1=jT+n and n_0=jT have been applied to Eq. 1.

The distribution function of the residual strength R(jT+n) can be obtained from that of R(jT) through the relation of Eq. 30 as

$$F_{R(jT+n)}(x) = P[R(jT+n) \leq x] = P[R^c(jT)-\Phi n \leq x^c]$$

$$= P\left[R(jT) \leq (x^c+\Phi n)^{1/c}\right]$$

Substitution of Eq. 28 into the above equation yields

$$F_{R(jT+n)}(x) = 1-\exp\left\{\left(\frac{r_0^c+\Phi jT}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c+\Phi(jT+n)}{\beta^c}\right)^{\alpha/c}\right\} \tag{31}$$

for the distribution function of the residual strength $R(jT+n)$.

The fatigue life after jT cycles, denoted by N_j, can be obtained from Eq. 30 by use of the fatigue failure condition, i.e., $n=N_j$, $R(jT+n)=\sigma_{max}$, as follows;

$$N_j = [R^c(jT)-\sigma_{max}^c]/\Phi \tag{32}$$

Therefore, the distribution function of the fatigue life after jT cycles can be obtained from that of R(jT), given by Eq. 28, through the transformation of Eq. 32, as follows;

$$F_{N_j}(n) = P[N_j \leq n] = P[R^c(jT) \leq n\Phi+\sigma_{max}^c]$$

$$= P\left[R(jT) \leq (n\Phi+\sigma_{max}^c)^{1/c}\right]$$

$$= 1-\exp\left\{\left(\frac{r_0^c+\Phi jT}{\beta^c}\right)^{\alpha/c} - \left(\frac{n\Phi+\sigma_{max}^c+\Phi jT}{\beta^c}\right)^{\alpha/c}\right\} \tag{33}$$

in which Φ is given by Eq. 29.

It can be observed that the distribution functions of the fatigue life, Eq. 33, and the residual strength, Eq. 28, reduce to Eqs. 22 and 23, respectively, when j=0, in which case the specimen is subjected to an initial high load r_0 only.

SECTION IV

EXPERIMENTAL VERIFICATION

1. SPECIMEN DESCRIPTION AND TEST PROCEDURE

The graphite/epoxy composite material used in this study is the Rigidite 5208-Thornel 300 system. Three 2'X2' panels were cured using an autoclave at the Air Force Materials Laboratory. The panels were post-cured for four hours at 400°F and the lay-up of the laminates is $[90/45/-45/0]_s$.

Test specimens were cut from the panels and numbered. Each specimen is about 8 in. long and 1 in. wide. The variation in width is 0.997"$\pm$0.007" and that in thickness is 0.046"$\pm$0.003". Nontapered glass/epoxy end tabs of 1.5 in. long were used.

Static tension tests were conducted on an MTS servo-hydraulic test machine and the loading rate was controlled manually. The initial high load prior to the fatigue tests was also performed manually.

Fatigue tests were conducted in room temperature with constant amplitude, tension-tension sinusoidal load cycles. In all tests, the R-value ($\sigma_{min}/\sigma_{max}$) was chosen to be 1/36 and the load frequency was 10 Hz.

2. Test Results

A set of fifteen (15) specimens were tested statically until fracture. The results of the ultimate strengths are presented in Table 1. The two-parameter Weibull distribution, Eq. 4, is used to fit the ultimate strength data. The shape

parameter α and the scale parameter β have been estimated as

$$\alpha = 25.35, \ \beta = 88.78 \text{ ksi } (612.1 \text{MN/m}^2) \qquad (34)$$

The fitted Weibull distribution is plotted as a solid curve in Fig. 1, along with the ultimate strength data plotted as circles.

A set of thirty nine (39) specimens were fatigue tested under various cyclic stress amplitudes. These results, referred to as the fatigue scan data, are given in Table 2. In Table 2, twenty five (25) specimens failed in fatigue, and the residual strengths of the remaining 14 specimens were measured.

The ultimate strength data (Table 1) and the fatigue scan data (Table 2) are used to determine the parameter values c,b and K using the analysis procedure described previously; with the results,

$$c = 12.2, \ b = 14.98, \ K = 7.49\text{X}10^{-32} \qquad (35)$$

where the unit of the strengths R(n) and R(0) as well as the applied stress range S is in ksi.

The fatigue scan data presented in Table 2 have been converted into the equivalent ultimate strengths using Eq. 26, along with the values of c,b and K determined in Eq. 35. The equivalent ultimate strengths are then plotted in Fig. 3 as circles, along with a solid curve that represents the same Weibull distribution presented in Fig. 2.

It is observed from Fig. 3 that the correlation between the equivalent ultimate strengths converted from the fatigue scan

data and the theoretical Weibull distribution is reasonable although it is not outstanding. It should be mentioned that the mean and the standard deviation of the Weibull distribution (solid curve) have been matched extremely close to those of the equivalent ultimate strengths in Eqs. 24 and 25. However, the skewness is not well matched. Such a discrepency is due to the poor quality of the specimens resulting from the manufacturing process and it will be anticipated later in other test results.

3. Theoretical and Experimental Correlations

After the parameter values c,b and K have been determined from the ultimate strength data (Table 1) and the fatigue scan data (Table 2), the theoretical model can predict the statistical distributions of the fatigue life and the residual strength as represented by Eqs. 9 and 10. Furthermore, the theoretical model is capable of predicting the effect of the high load on the fatigue behavior of composite laminates as given by Eqs. 18, 22 and 23.

To verify the validity of the theoretical model, a set of twenty (20) specimens were tested at a maximum cyclic stress of 60.57 ksi (417.6 MN/m^2) until fatigue fracture. The results are presented in Table 3 and plotted in Fig. 4 as circles. The theoretical prediction for the statistical distribution of the fatigue life represented by Eq. 9 is also plotted in Fig. 4 as a solid curve for comparison. It is observed from Fig. 4 that the correlation between the theoretical prediction (the solid curve) and the test results (circles) is outstanding.

A set of sixteen (16) specimens were subjected to a maximum cyclic stress of 56.53 ksi (389.76 MN/m^2) for 56,000 cycles, and then the residual strengths were measured. The test results are presented in Table 4. The probability that a specimen may fail before 56,000 cycles of load application is computed from Eq. 9 as 0.204. Hence, the expected number of specimens that will fail before 56,000 load cycles is 0.24x16 = 3.27. Test results indicate no failure at all [see Table 4]. This may be caused by the following two reasons; (i) The quality of the specimens are poor due to the manufacturing process thus resulting in a serious sampling fluctuation as anticipated previously. Hence, the reproduciability is poor. (ii) it is observed from Fig. 3 that, in the lower tail region of the distribution, the theoretical Weibull distribution (solid curve), which is used as the basis for the theoretical prediction, predicts more early failures than the actual fatigue test results (circles). In other words, the weak specimens in fatigue are stronger than the ones predicted by the Weibull distribution.

The residual strength data (Table 4) are plotted as circles in Fig. 5 along with the solid curve that represents the theoretical prediction of the residual strength distribution constructed using Eq. 10. It is noticed from Fig. 5 that the theoretical prediction is conservative in the lower tail portion of the distribution function and it becomes slightly unconservative (over estimate the residual strength) after the 25% data point.

The fact that the theoretical model is conservative in the lower tail portion of the distribution function is beneficial, since the lower tail region of the distribution is more relevant to the practical design. It is further observed from Fig. 5 that even in the region greater than the 25% point, the theoretical prediction is qualitatively reasonable as explained in the following; (i) The maximum discrepency between the theoretically predicted distribution (solid curve) and the test results (circles) is approximately 0.05β (which occurs near the 50% point), which is only about 5.4% of the mean residual strength, (ii) ten (10) out of a total of sixteen (16) data points have a residual strength between 0.8β and 1.0β, which corresponds to the region between 25% and 90% of the theoretical distribution (solid curve). This indicates that $10/16 \simeq 62.5\%$ of the test data falls into the region (0.8β to 1.0β) in which the theoretical prediction is 90%-25%=65%. Consequently, 62.5% of the test results having a residual strength between 0.8β and 1.0β compares favorably with the theoretical prediction that is 65%.

In view of these observations, the correlation between the theoretically predicted distribution and the test results appears to be acceptable although a slight quantitative discrepency does exist in the region greater than the 25% point of the distribution function.

In order to verify the validity of the theoretical predictions for the effect of the high load on the fatigue behavior of

composites as derived in Eqs. 18,22, and 23, fourty six (46) specimens were subjected to a high load r_0 = 82.64 ksi (569.78 MN/m^2) prior to the fatigue loading. Under the high load r_0, eight (8) specimens failed as shown in Table 5(c). The probability of failure under a high load of 82.64 ksi (569.78 MN/m^2) is computed from Eq. 4 as 0.15. Hence, the expected number of specimens that will fail under r_0 is 0.15x46 $\simeq$ 7. Hence, the theoretical prediction (7 failures)compares favorably with the test results that 8 specimens fail. Then, the thirty eight (38) specimens, which survived the high load, were separated into two sets (19 specimens each) for fatigue tests.

The first set of 19 specimens was subjected to the cyclic loading at a maximum stress of 64.9 ksi (447.47 MN/m^2) until fracture. The results are given in Table 5(a). According to the theoretical prediction represented by Eq. 18, a specimen which survives the high load r_0=82.46 ksi (569.78 MN/m^2) is guarranteed to sustain at least n_0=5,730 cycles of load application. Test results presented in Table 5(a) indicate that the fatigue life of every specimen is greater than 5,730 cycles. This is an indication that the survival of a high load indeed will guarrantee a fatigue life N_0 as predicted theoretically by Eq. 18.

Test results on the fatigue life are plotted in Fig. 6(a) as circles. Also plotted in Fig. 6(a) as a solid curve is the theoretical prediction constructed by use of Eq. 22. It is observed from Fig. 6(a) that the correlation between the

theoretical distribution and the test results is very reasonable in the lower tail portion of the distribution function, which is of particular importance to the design of aircraft structures. The theoretical distribution, however, slightly underestimates the fatigue life of the test results in the region greater than the 30% point, indicating that the theoretical model is slightly conservative in predicting the fatigue life having the effect of the high load.

An examination of the fatigue data presented in Table 5(a) indicates that the highest four (4) data points lie between 35,000 cycles to 39,000 cycles. These four data points do not seem to belong to the same population as the rest of the data points, because they correspond to the 55% point of the fatigue life for specimens under a maximum cyclic stress of 60.57 ksi (417.6 MN/m^2) as indicated by Fig. 4 and Table 3. It is inconceivable that the fatigue life under a maximum cyclic stress of 64.9 ksi (447.47 MN/m^2) will fall into the central region (55%) of that under a maximum cyclic stress of 60.57 ksi (417.62 MN/m^2). The only explanation is that the specimens are not manufactured uniformly with good quality control. For the purpose of comparison, the test results of the fatigue life are plotted in Fig. 6(b) with the exclusion of these four data points. The solid curve plotted in Fig. 6(b) is the theoretical prediction represented by Eq. 22, which is identical to the solid curve presented in Fig. 6(a). It can be observed from Fig. 6(b) that the correlation between the theoretical prediction and the test results is outstanding.

The second set of 19 specimens was subjected to a maximum cyclic stress of 60.57 ksi (417.6 MN/m^2) for 20,000 cycles and then their residual strengths were measured. The results are presented in Table 5(b). According to the theoretical model, Eq. 18, the specimen which doesn't fail under the high load r_0= 82.46 ksi (569.78 MN/m^2) will survive at least 16,730 cycles of load application. Since no specimen fails before 20,000 load cycles (see Table 5(b)) it further confirms the fact that the survival of a high load will guarrantee a fatigue life that can be predicted by the theoretical model represented by Eq. 18.

Test results are plotted in Fig. 7 as circles along with the solid curve which represents the theoretical distribution constructed using Eq. 23. The correlation between the theoretical prediction and the test results is similar to that of the situation without the application of a high load as presented in Fig. 5. In other words, the theoretical prediction is slightly conservative in the lower tail portion of the distribution function while it cverestimates slightly the residual strength in the region greater than the 15% point. It is observed from the solid curve (theoretical model) of Fig. 7 that the probability that the residual strength lies between 0.85β and 1.0β is 70% (i.e., in the region between 20% and 90% of the distribution function). It is further observed from Fig. 7 that the residual strengths of 13 data points out of a total of 19 data points lie between 0.85β and 1.0β. As a result, the percentage of data

points having a residual strength between 0.85β and 1.0β is 13/19~68.4% as compared to 70% predicted by the theoretical model. Hence the correlation between the test results and the theoretical model in the central region of the distribution function appears to be qualitatively reasonable, although there is a slight quantitative discrepency. The fact that the theoretical prediction is slightly conservative in the lower tail portion of the distribution function, which is of practical importance, is beneficial.

SECTION V

CONCLUSION AND DISCUSSION

A three-parameter fatigue and residual strength degradation model [Refs. 3,5] has been used to predict the effect of the high load on the fatigue behavior of unnotched composite laminates. The number of load cycles a specimen can survive, after it has survived a certain level of high load, and the statistical distributions of the fatigue life and the residual strength under the subsequent load application have been derived.

An experimental test program has been carried out to verify the theoretical model, as well as the theoretical prediction for the effect of the high load on the fatigue behavior of unnotched graphite/epoxy laminates.

Based on the results of the present test program several observations are summarized in the following:

1) The three-parameter fatigue and residual strength degradation model, referred to as the theoretical model, is reasonable as compared to the test results.

2) The most important effect of the high load on the fatigue behavior of composites is that a specimen can survive at least a certain number of load cycles after it has survived a ceratin level of high load prior to the fatigue loading [Refs. 1-3]. This effect has been confirmed by the present test program. In fact, our test results indicate that such an effect can be predicted from the theoretical model.

3) The theoretical model for predicting the statistical distributions of the fatigue life and the residual strength under the application of a high load followed by the cyclic loading appears to be acceptable as compared to the test results.

4) The theoretical prediction for the statistical distributions of the fatigue life and the residual strength is slightly conservative in the lower tail protion of the distribution function as compared to the test results. This is a beneficial trend, since the lower tail portion of the distribution is of practical importance in the fatigue design of aircraft structures. However, this trend is not conclusive. In contrast to this trend, the test results generated in Ref. 6 and analyzed in Ref. 5 indicates that some outliers do exist in the lower tail portion of the distribution function.

5) While the theoretical prediction for the statistical distribution of the fatigue life has a good correlation with the test results, the theoretical prediction for the residual strength is slightly unconservative in the central portion of the distribution function as compared to the test results. Again, this observation is not conclusive, since the trend does not exist for the test results generated in Ref. 6 and analyzed in Ref. 5.

We have experienced a considerable sampling fluctuation in the present test program as discussed in the previous section. This may be attributed to the poor quality control in

manufacturing the specimens, including curing, stacking of prepegs, fabrication, and machining. Since an excessive sampling fluctuation gives rise to the unpredictability of the fatigue behavior of composites, it is important that the specimens with good quality control should be used when the test program is intended to verify a theoretical model.

One of the important advantages of the three-parameter theoretical model discussed herein is the proposed method of analysis for the determination of the three model parameters using only a limited amount of test data [Refs. 3,5]. As described previously, the tests one has to perform consist of one set of the static ultimate strength data (approximately 15∿25 specimens) and one set of fatigue scan data (approximately 30∿40 specimens) only. Once the parameter values c,b and K have been determined from these two sets of data, the statistical distributions of both the fatigue life (under any cyclic stress level) and the residual strength (for any number of cycles of load application) can be predicted theoretically [see Eqs. 9 and 10]. Moreover, without any additional test data, the theoretical model can predict the effect of the high load on the fatigue behavior of composite laminates, including (i) the minimum number of load cycles a specimen can survive after it passes a high load [see Eq. 18] and (ii) the statistical distributions of the fatigue life and the residual strength for the specimens subjected to a high load followed by the fatigue loading.

On the basis of the present research program, including the theoretical investigation and data analyses of the test results

presented in Ref. 5, as well as the past experiences [Ref. 2], it is concluded that the three-parameter fatigue and residual strength degradation model is reasonable in predicting the gross fatigue behavior of unnotched composite laminates. The fatigue behavior includes the statistical distributions of the fatigue life and the residual strength, as well as the effect due to the application of high loads. This conclusion is based on the fact that no significant discrepency has been observed in the correlation between the theoretical model and the extensive amount of test results.

It should be mentioned that a theoretical model which can predict accurately the fatigue life and the residual strength of composites in the entire region of the statistical distribution may be very difficult to establish, since the fatigue behavior of composites is a complex process influenced by so many variables. In view of the complexity of the fatigue process involved, the three-parameter fatigue and residual strength degradation model seems to offer a simple and reasonable approach for the preliminary fatigue analysis.

Finally, the theoretical model discussed herein is also applicable to the case when the fatigue failure is governed by the matrix failure mode [Ref. 13].

REFERENCES

1. Hahn, H. T., and Kim, R. Y., "Proof Testing of Composite Materials," Journal of Composite Materials, Vol. 9, 1975, p. 297.

2. Halpin, J. C., et al., "Characterization of Composites for the Purpose of Reliability Evaluation," Air Force Materials Laboratory Technical Report AFML-TR-77-289, 1972.

3. Yang, J. N., and Liu, M. D., "Residual Strength Degradation Model and Theory of Periodic Proof Tests for Graphite/Epoxy Laminates," Journal of Composite Materials, Vol. 11, 1977, p. 176.

4. Yang, J. N., "Reliability Prediction of Composites Under Periodic Proof Tests in Service," Composite Materials: Testing and Design, ASTM-STP #617, 1977, p. 272.

5. Yang, J. N., "Fatigue and Residual Strength Degradation for Graphite/Epoxy Composite Under Tension-Compression Cyclic Loadings," Journal of Composite Materials, Vol. 12, 1978, pp. 19-39.

6. Ryder, J. T., and Walker, E. K., "Ascertainment of the Effect of Compressive Loading on the Fatigue Life Time of Graphite/Epoxy Laminates For Structural Application," AFML-TR-76-241, WPAFB, December 1976.

7. Hahn, T. H., "Fatigue Behavior and Life Prediction of Composite Materials," paper presented at the ASTM 5th Conference on Composite Materials; Testing and Design, March 20-22, 1978, New Orleans.

8. Halpin, J. C., Johnson, T. A., and Waddoups, M. E., "Kinetic Fracture Models and Structural Reliability," Int. Journal of Fracture Mechanics, Vol. 8, 1972, p. 465.

9. Manning, S. D., Lemon, G. H., Waddoups, M. E., and Achord, R. T., "Composite Wing for Transonic Improvement, Vol. III-Structural Reliability Study," Air Force Flight Dynamic Laboratory Technical Report, AFFDL-TR-71-24, November 1972.

10. Wolff, R. V., and Lemon, G. H., "Reliability Prediction for Adhesive Bonds," Air Force Materials Laboratory Technical Report, AFML-TR-72-21, 1972.

11. Wolff, R. V., and Lemon, G. H., "Reliability Prediction for Composite Joints," Air Force Materials Laboratory Technical Report, AFML-TR-74-197, 1974.

12. Chou, P. C., "Degradation and Sudden-Death Model of Fatigue of Graphite/Epoxy Composites," paper presented at the ASTM 5th Conference on Composite Materials; Testing and Design, March 20-22, 1978, New Orleans.

13. Yang, J. N., and Jones, D. L., "Fatigue of Graphite/Epoxy Composite in Shear," paper presented at the ASTM 5th Conference on Composite Materials; Testing and Design, March 20-22, 1978, New Orleans.

TABLE 1. STATIC TENSILE STRENGTH

SPECIMEN NO.	ULTIMATE STRENGTH R(0), ksi
1-1-7	84.809
1-1-13	82.541
1-1-20	81.199
1-1-26	84.793
1-1-33	87.494
2-1-3	80.930
2-1-11	87.656
2-1-27	92.957
2-2-1	86.546
2-2-14	87.054
3-1-9	90.565
3-2-23	83.676
3-1-28	95.978
3-1-14	90.589
3-2-10	86.523
Average	86.887
α	25.353
β	88.777

TABLE 2. FATIGUE SCAN RESULT
($\sigma_{min} = \sigma_{max}/36$)

SPECIMEN NO.	MAXIMUM STRESS σ_{max} (ksi)	FATIGUE LIFE (10^3 CYCLES)	RESIDUAL STRENGTH (ksi)**
1-1-6	71.600	1.0	
3-2-1	"	1.33	
2-2-7	"	2.43	
1-1-8	67.330	6.09	
3-2-31	"	5.23	
3-1-33	"	22.38	
2-2-16	"	25.30	
1-1-23	63.100	24.53	
3-1-19	"	42.88	
3-1-1	"	8.52	
2-1-5	"	20.49	
2-1-5	"	21.42	
1-1-2	58.920	27.69	
1-1-15	"	30.32	
3-2-24	"	126.59	
2-1-14	"	60.15	
2-1-29	"	103.58	
2-2-2	"	125.79	
2-1-22	57.240	137.42	
1-2-15	"	56.13	
1-1-28	54.700	589.19	
3-1-15	"	133.39	
1-2-23		155.27	
1-2-21		154.6	

(Continued)

TABLE 2. FATIGUE SCAN RESULT (CONTINUED)
($\sigma_{min} = \sigma_{max}/36$)

SPECIMEN NO.	MAXIMUM STRESS σ_{max} (ksi)	FATIGUE LIFE (10^3 CYCLES)	RESIDUAL STRENGTH (ksi)**
2-1-24	50.500	454.11	
1-2-1	58.920	20,000*	78.696
2-2-3	"	20,000*	70.619
1-2-5	63.120	7,000*	80.185
2-2-15	"	7,000*	75.057
1-1-3	56.524	56,000*	64.39
2-1-1	56.524	56,000*	64.18
1-1-34	56.524	56,000*	68.40
3-2-9	56.524	56,000*	62.98
2-1-2	56.524	56,000*	69.11
2-2-36	56.524	56,000*	69.38
1-2-20	56.524	56,000*	70.29
3-1-32	56.524	56,000*	71.14
2-1-4	54.710	100,000*	71.414
3-2-20	"	100,000*	63.608

* = SPECIMEN DOES NOT FAIL IN FATIGUE

** = RESIDUAL STRENGTH AT FATIGUE FAILURE IS EQUAL TO σ_{max}

TABLE 3. FATIGUE LIFE TEST RESULT
$\sigma_{max} = 60.567$ ksi, $\sigma_{min} = \sigma_{max}/36$

SPECIMEN NO.	FATIGUE LIFE (CYCLES)
1-1-4	15,080
2-2-10	17,960
3-2-18	22,000
2-2-4	23,390
2-1-19	23,980
2-1-31	25,000
2-2-21	25,900
1-2-33	27,340
3-2-22	32,870
3-2-2	37,860
1-1-32	39,540
1-2-25	40,680
3-1-18	43,510
3-1-31	46,080
1-1-12	49,440
3-1-26	51,700
2-2-32	52,510
1-2-18	70,450
1-2-2	72,630
2-1-7	135,500

TABLE 4. RESIDUAL STRENGTH TEST RESULT
σ_{max} = 56.53 ksi, n = 56,000 cycles
$\sigma_{min} = \sigma_{max}/36$, β = 88.78 ksi

SPECIMEN NO.	RESIDUAL STRENGTH (ksi)
2-2-5	65.63
2-2-33	68.61
2-2-12	68.96
2-2-20	69.80
1-1-30	70.71
2-1-8	71.89
1-1-18	73.53
1-2-27	74.08
3-2-3	75.92
3-2-17	79.25
3-1-24	81.11
3-1-27	81.27
1-2-12	81.37
3-1-16	84.98
3-2-6	85.22
1-2-16	89.77

TABLE 5. FATIGUE LIFE AND RESIDUAL STRENGTH TEST RESULT
HIGH LOAD r_0 = 82.64 ksi

(a) σ_{max} = 64.9 ksi		(b) σ_{max} = 60.57 ksi, n = 20,000 cycles	
SAMPLE NO.	FATIGUE LIFE CYCLES	SAMPLE NO.	RESIDUAL STRENGTH (ksi)
1-2-13	7,550	1-1-10	68.69
2-1-26	8,000	3-2-4	72.12
3-1-29	8,060	2-2-6	73.71
2-1-25	8,390	1-2-24	73.76
2-2-19	10,250	3-1-7	74.55
3-1-23	10,500	2-2-23	74.75
2-1-13	14,250	1-2-26	76.86
2-1-17	14,500	3-1-22	77.41
2-1-33	16,240	2-2-8	77.47
2-2-9	17,450	1-2-17	77.66
1-1-27	19,330	3-2-8	78.42
1-1-11	22,040	1-2-14	78.80
1-2-22	22,890	3-1-6	79.89
3-1-25	25,630	1-2-7	80.11
2-2-29	26,730	1-2-13	82.08
1-2-6	35,000	3-1-11	82.87
3-1-30	37,080	1-1-16	85.29
1-2-8	37,750	2-1-18	85.95
2-2-17	39,240	3-1-13	88.42

(c) STATIC FAILURE UNDER HIGH LOAD r_0

SAMPLE NO.	STRENGTH (ksi)	SAMPLE NO.	STRENGTH (ksi)
2-2-18	75.24	2-2-28	81.02
1-2-26	76.86	1-1-30	81.73
1-1-25	78.80	3-2-25	82.04
2-1-6	80.47	2-2-31	82.44

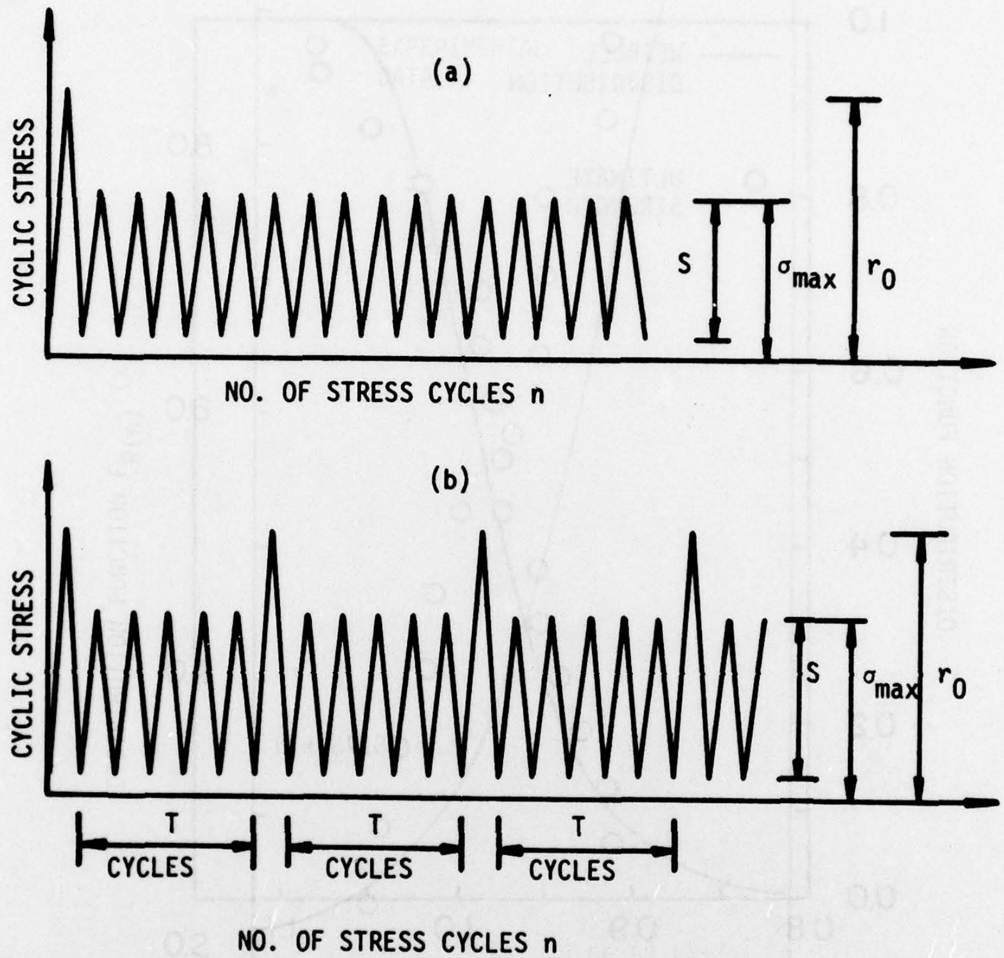

Fig. 1. Loading history with (a) initial high load (b) periodic high loads.

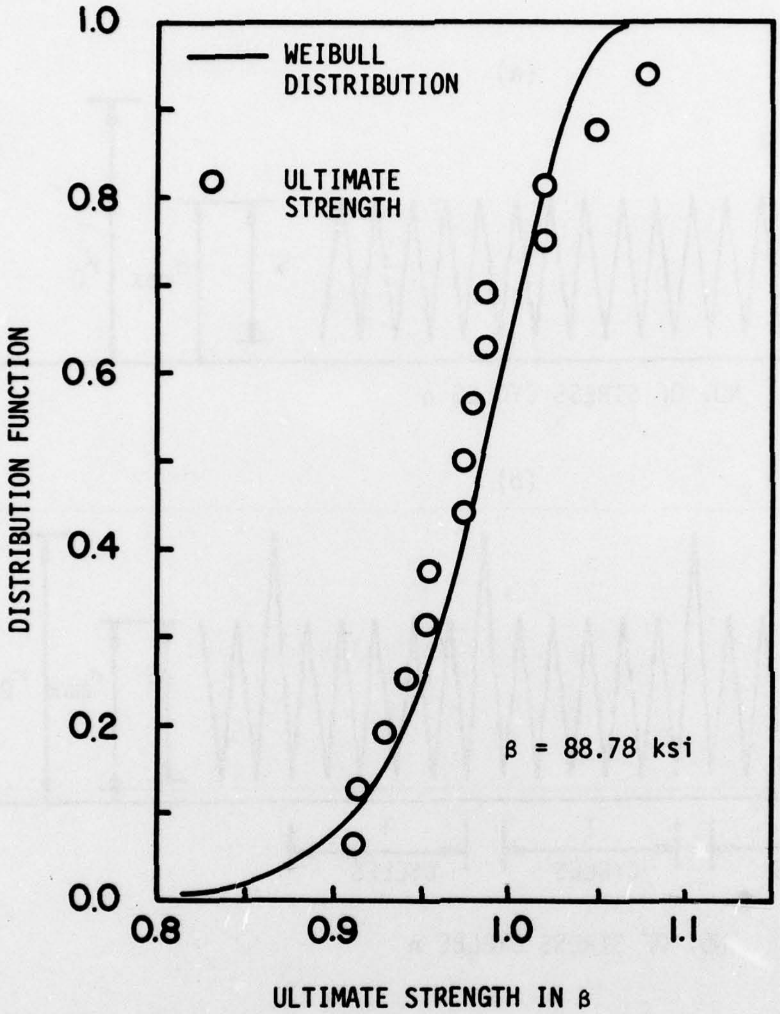

Fig. 2. Distribution of the static ultimate strength as a function of β and the associated two-parameter Weibull distribution.

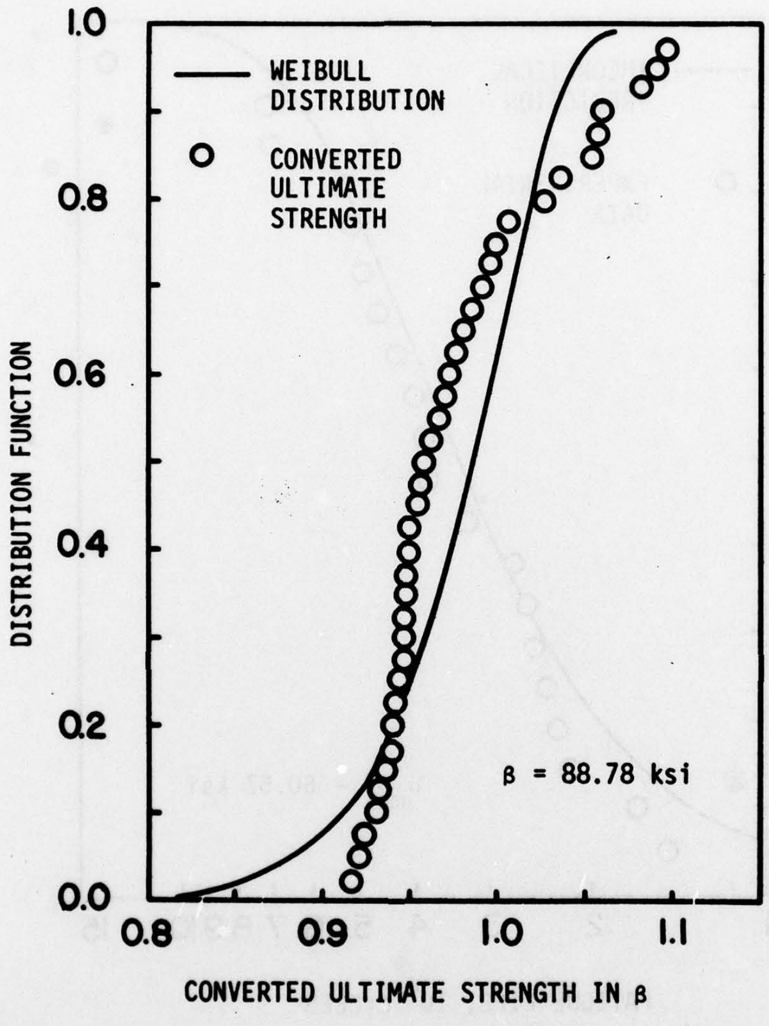

Fig. 3. Distribution of the converted ultimate strength from the fatigue scan data presented in Table 2.

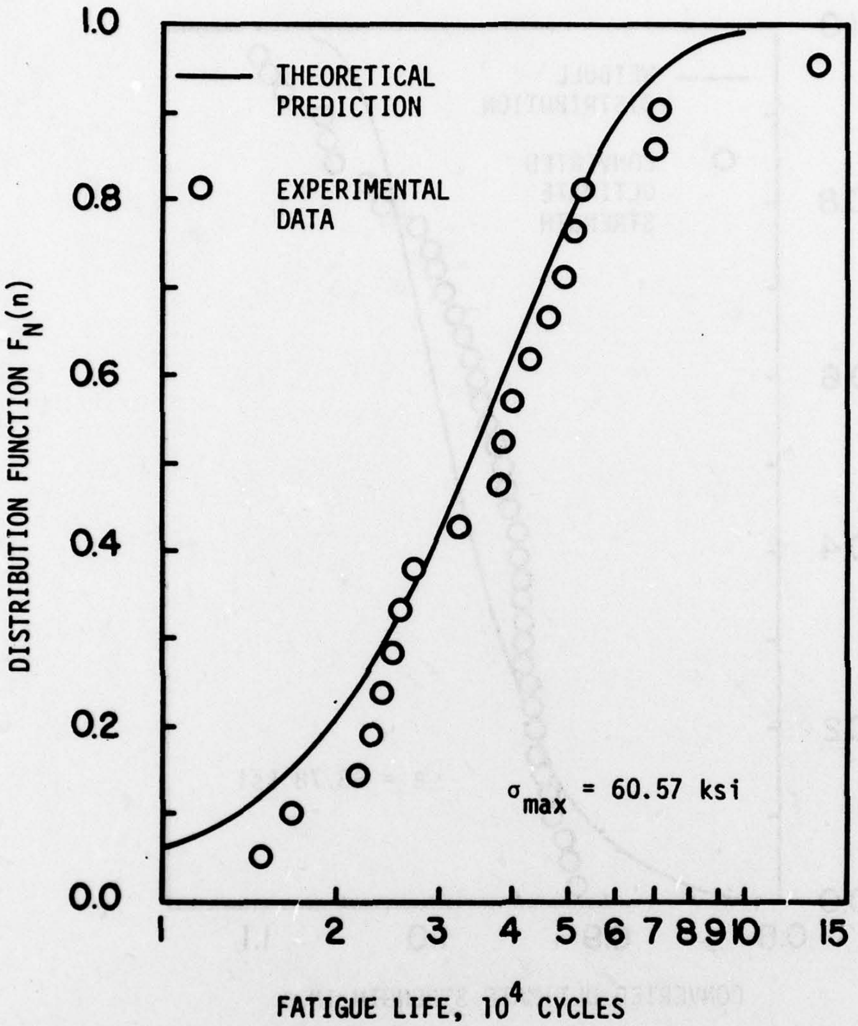

Fig. 4. Comparison between the theoretically predicted distribution of fatigue life and the test data.

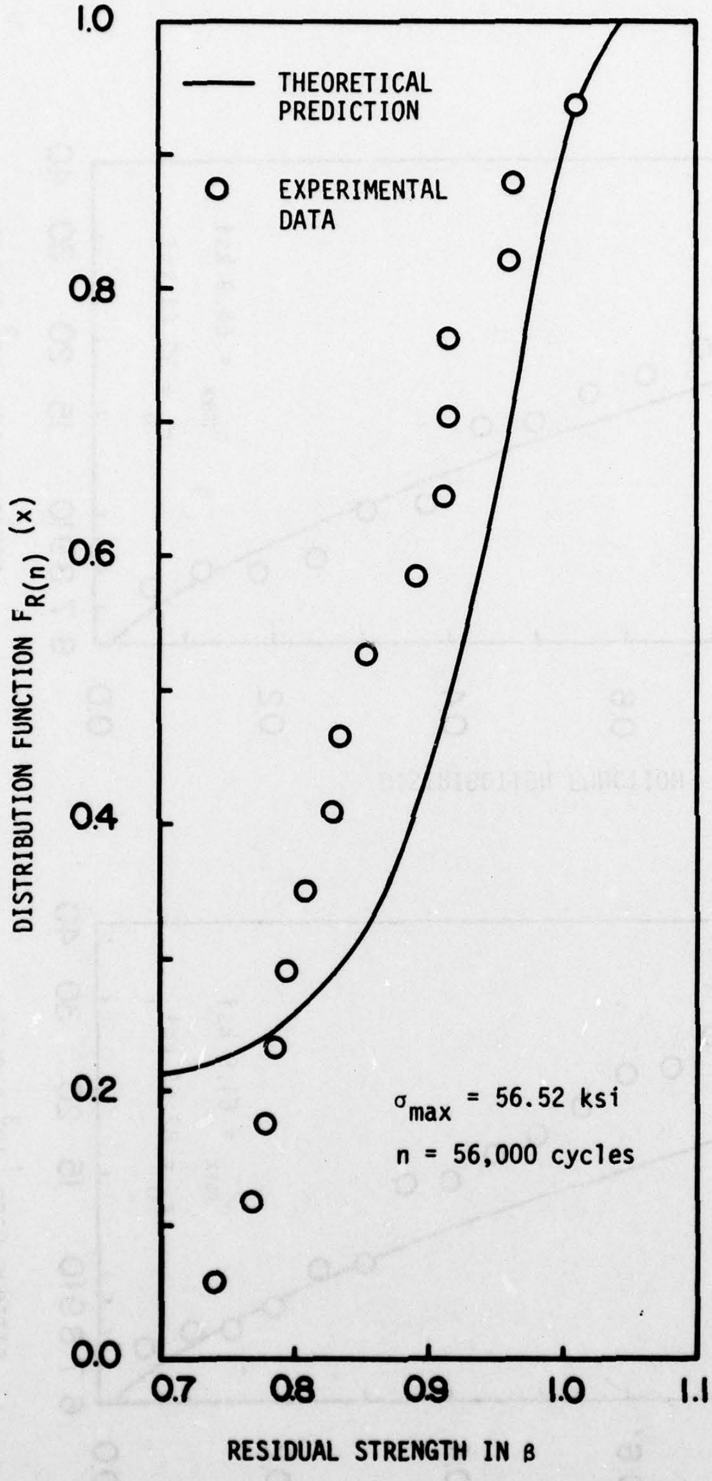

Fig. 5. Comparison between the theoretically predicted distribution of the residual strength and the test data.

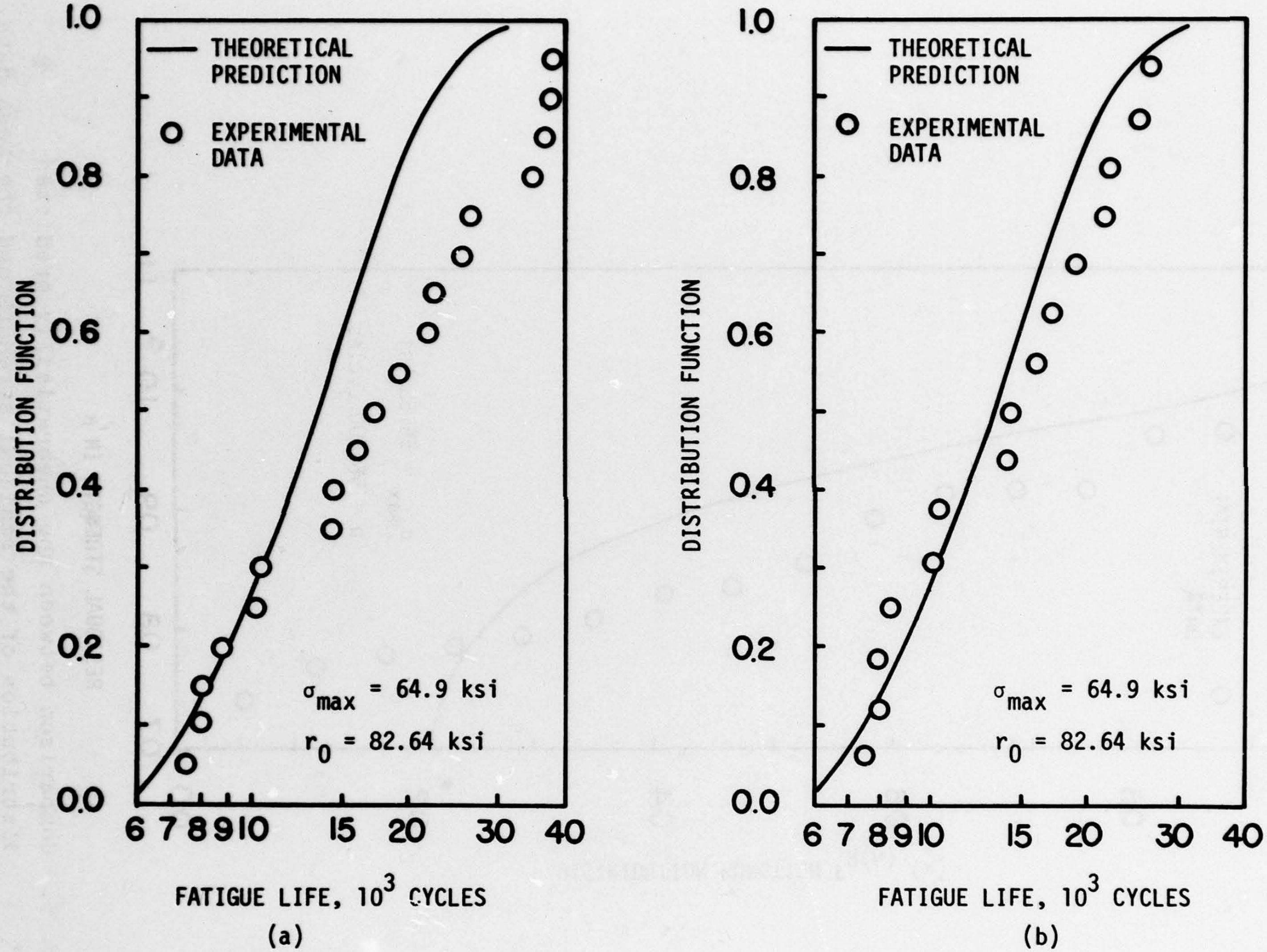

Fig. 6. Comparison between the theoretically predicted distribution of the fatigue life and the test data including the effect of high load.

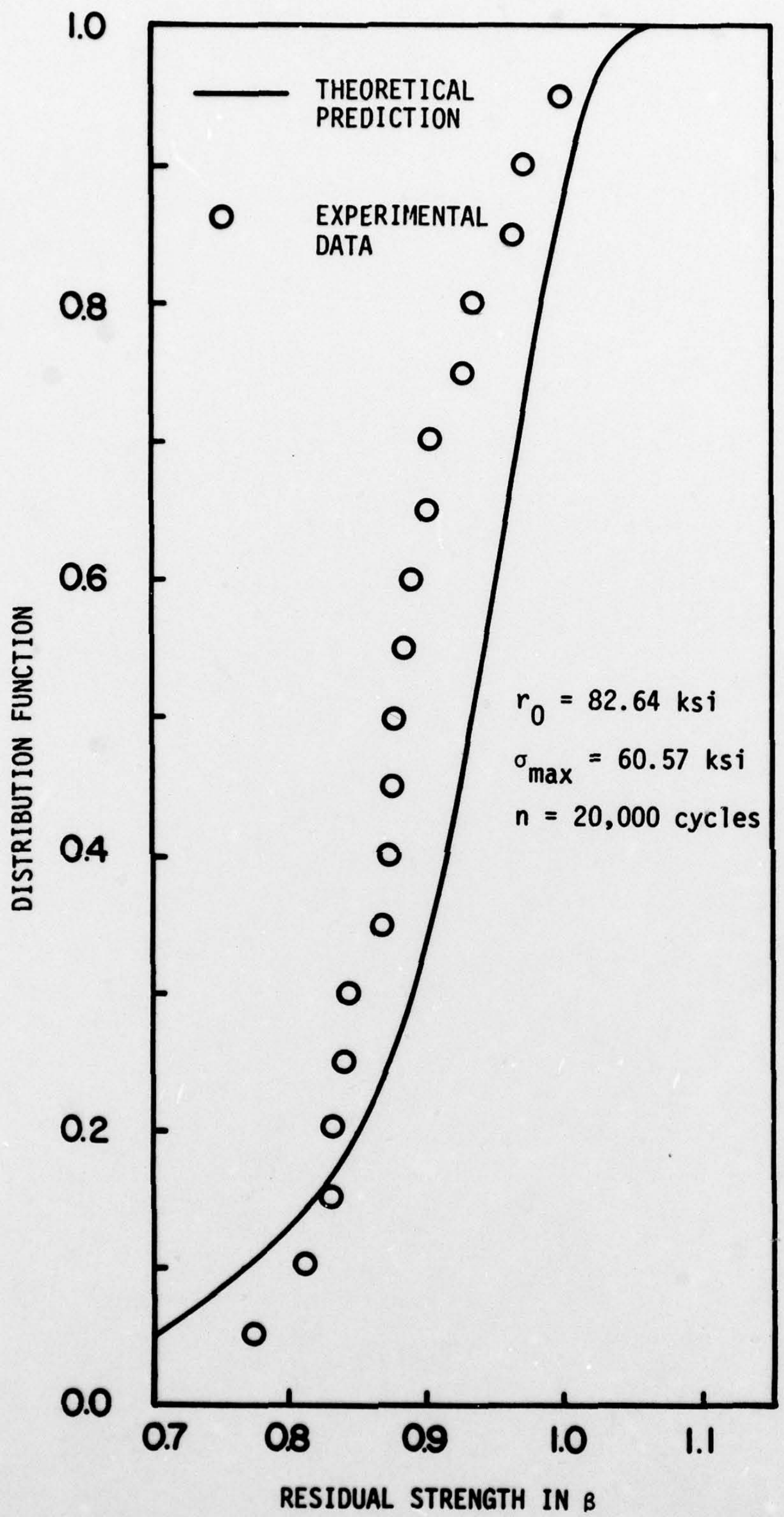

Fig. 7. Comparison between the theoretically predicted distribution of the residual strength and the test data including the effect of high load.

*U.S.Government Printing Office: 1979 — 657-002/448